**Charge to Mass Ratio of the Electron**

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**Abstract:**

With the help of a device able to eject particles and transfer electricity through a gas, an electron gun contained in a vacuum tube filled with helium, my lab partner and I were able to obtain a value for the charge to mass ratio of an electron (e / m). This lab calculated the values of E/M through two different methods that should nearly obtain the same result. The first of these two methods included a varying current supply for the Helmholtz coil and a constant voltage for the electron gun. The second of these two methods included a varying power supply for the electron gun and a constant current for the Helmholtz coil. The measured value of charge to mass ratio (e / m) was concluded to be (2.5 ± 0.9) x 1011 C/kg for a changing amperage. The measured value of charge to mass ratio (e / m) was concluded to be (1.6 ± 0.1) x 1011 C/kg for a changing voltage. Of these two values, my lab partner and I agree the second value, (1.6 ± 0.1) x 1011 C/kg, is better representative of our lab tests because the data has a much better linear fit. The ratio we achieved is within the calculated uncertainty range of the theoretical value, (1.75882017 +/– 0.00000007) x 1011 C/kg. The difference in uncertainty values from this labs conclusion to the accepted theoretical value is a result of experimental error. Error was rampant in this lab. Error accumulated as a result of poor lighting, multiple reflections, and a wide beam even at its smallest, lab technicians’ poor eyesight, and a difficult system for measuring diameters.

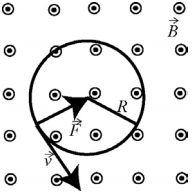
**Introduction & Theory:**

Under the ideas denoting electricity is carried in gases by particles, this lab will produce an observable electron beam curved through the power of magnetic fields. With the use of a simple ruler, we can measure the radius of an electron’s orbit, and effectively calculate an estimate of its mass, but only with the use of correct data. The data over a series of calculations may produce a large uncertainty, but this is because uncertainties from various entities will accumulate over time. Yet, even with even remotely fair data, one can achieve a reasonably accurate ratio of electric charge to mass for the electron.

When a filament reaches a certain point in heat, it can begin to produce free electrons within the apparatus or the vacuum tube. These electrons are then accelerated through a varying electric potential V with the help of an electric field produced by the apparatus. These electrons will follow the suit of energy laws and relations, specifically regarding kinetic energy.

½mv2 = eV (1)

Equation 1 relates half of the product of the mass of an electron (m) and its velocity (v) squared with the product of the charge of an electron (e) and the potential difference (V).

 According to known theory, the electrons will move with perspective from corresponding electrostatic or magnetic fields. As seen in the diagram to the left, we can observe the path of the electron in the diagram as counterclockwise, due to an into-the-page and perpendicular magnetic field. The force felt by this electron is given as follows:

F = -e(v *x* B) (2)

Equation 2 relates the transverse force (F) onto an electron with the negative electric charge (e) of an electron and the cross product between the velocity vector (v) and the magnetic field vector (B).

According to Equation two and by confirmation of the right hand rule, the force onto the electron is perpendicular to the direction of the magnetic field and to the velocity of the electron. Because this force pulls in a perpendicular direction at all times to the velocity vector, the velocity will begin to change and the electron will project circular motion.

F = evB (3)

evB = mv2 / R (4)

When considering the use of an electron, the magnitude of the magnetic force can be represented by the products of the charge of the electron (e) and the velocity (v) and the magnetic field strength (B) as done in equation 3. This force is opposing a centripetal force, so sum of the two forces must be zero because the circle is unchanging, thus, equation 4 comes into fruition; the mass of the electron (m) and its squared velocity (v) divided by its radius (R) will balance out the magnetic field’s force onto the electron.

Further simplifying, we can obtain a value for the ratio of charge to mass of the electron:

e / m = (2V) / (BR)2 (5)

We should hope to find a value of e / m as close to the modern accepted value for the ratio as possible: (1.75882017 ± 0.00000007) x 1011 C/kg. However, due to time limitations, and the precision and accuracy of the tools and our judgement used, we will not be able to conclude with such an accurate measurement. However, the limits of this experiment constrain us into an inability to calculate the charge and mass directly. One can use other relations to uncover a specific value for the charge. One such method to get there is to further examine our equation. Magnetic field (B) is dependent on the aspects decided from the Helmholtz coil.

B = 8μ0NIc / (5r sqrt(5)) (6)

Equation 6 includes variables where μ0 is a constant and equals 4π×10-7 T⋅m/A, Ic is the current in the coil, and N is the number of turns in each coil. This labs apparatus contained 130 turns, each with a radii of 0.150 ± 0.005 m. From here we can combine equations 5 and 6 to produce an equation independent of magnetic field B.

e / m = 2V / ((8μ0NIc / (5r sqrt(5))) R)2 (8)

Then we will rearrange for radius and its inverse.

1/R = \* Ic (9)

R = (10)

And if we account for an intercept to include external forces from magnetic fields, we obtain:

1/R = (9)

R = (10)

From here, we are now able to solve for the charge over mass ratio.

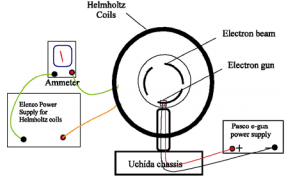
(13)

is the slope of the first graph.

(14)

is the slope of the second graph.

**Experimental Procedure:**

A vacuum tube in the shape of a giant bulb was filled with helium gas and mounted with an internal electron gun to produce the visible electron orbits that we had measured in this experiment. The vacuum tube was attached to an Uchida manufactured chassis, preassembled before lab. A Pasco power supply was then wired to supply a constant controlled voltage to the electron gun so may operate the apparatus. The object surrounding the vacuum tube, the thick dark circle in the figure to the right, is a Helmholtz coil which was given a controlled current by an Elenco supply. This lab began with adjusting this equipment to appropriate values. We adjusted the voltage power supply accordingly until we achieved the visible ring on electrons. This value occurred around 300 V of input electric potential form the Pasco supply unit. Then, using the current supply from the Elenco component, we adjusted the supply input until we achieved different sizes of the ring. Once there was a comfortable ring of decent size, the tuner could be used to adjust the width of the circles electron border. The tuner either increased the density of the electrons’ path or sprayed the electrons creating a “hazy” orbit. The Helmholtz coil is critically to this experiment. Without it, the electron gun would create a helix of electrons, too small for this class’s equipment to measure. The coil produces a large electric potential difference to increase the kinetic energy of the particle.

There were two parts to collecting data in this lab. Part one pertained to a constant current and varying voltage, while part two pertained to a constant voltage and a varying current. With both of these sections, we were instructed to measure the diameter of the loop with a reflective ruler placed behind the bulb. Measurements for the diameters needed to be precise and accurate. For accuracy, when taking a measurement, the lab technician must render the reflection hidden by placing himself a distance away from the apparatus and perpendicular to the ruler such that the actual electron ring overlaps with the mirrored image. With some larger ring diameters, this distance from the apparatus can get very large.

The first data set must be recorded such that voltage is held constant and current is varied. We found the maximum current supply and minimum current supply such that our ring stayed intact and did not devolve into a series of deflections within the bulb. We then proceeded to take five measurements of the ring size at varying magnitudes of current supply and recorded our results. There should be a noted change in size of the ring depending on the magnitude of current supplied.

The second data set must be recorded such that current is held constant and voltage is varied. We found the maximum voltage supply and minimum voltage supply such that our ring stayed intact and did not devolve into a series of deflections within the bulb. We then proceeded to take seven measurements of the ring size at varying magnitudes of voltage supply and recorded our results. There should be a noted change in size of the ring depending on the magnitude of voltage supplied.

**Results & Analysis:**

Note: Both sections for this experiment rely off a 0.5 cm uncertainty for my lab partner and I because this value represents our faith in our measurements and accounts for human error. These values were averaged between my partner and me to achieve greater accuracy.

|  |  |  |
| --- | --- | --- |
| Amperage (A) | Diameter (cm) | Radius-1 (m) |
| 1.49 ± 0.01 | 4.5 ± 0.5 | 44 ± 9 |
| 1.57 ± 0.01 | 4.0 ± 0.5 | 50 ± 10 |
| 1.60 ± 0.01 | 4.6 ± 0.5 | 43 ± 9 |
| 1.92 ± 0.01 | 3.9 ± 0.5 | 51 ± 13 |
| 2.21 ± 0.01 | 3.7 ± 0.5 | 54 ± 14 |
| 2.33 ± 0.01 | 3.5 ± 0.5 | 57 ± 16 |

For the first procedure, voltage was set to a constant 300 ± 1 V and the Elenco current supply began at 1.57 ± 0.01 A. We began to carry out the procedure and the data can be found to the right. The minimum amperage required to form the ring was 1.49 ± 0.01 A and the maximum amperage required to form the ring was 2.33 ± 0.01 A. Using the data found in this table, we entered it into origin, found values for the inverted radius and plotted these values as a function of amperage. The slope of the formed line was determined by origin to be 16 ± 3 m-1/A. This linear fit had an intercept of 20 ± 4 m-1/A, implying that our data was skewed. This value should be close to zero because as the supplied current is reduced, the radius of the electron ring should increase to an undefined number (radius = 1/0 m and radius inverted = 0/1 m-1). Our intercept value is unreasonable for this experiment as there is not a great enough external force to create such an error in results.

The uncertainty in the inverted radius can be found using the derivative method:

(15)

(16)

(17)

Using all of these values for the first test and equation 13, we calculated a value for the ratio:

(13)

From here, we can calculate the uncertainty in this value as a result of many contributors:

The final value for the results of the ratio in section one was (2.5 ± 0.9) x 1011 C/Kg. This value makes analytical sense as it is close to the currently accepted value. In fact, the accepted theoretical value for the ratio is within the uncertainty of our measured value. Therefore, our results can be deemed accurate.

|  |  |  |
| --- | --- | --- |
| Voltage (V) | Voltage1/2 (V) | Radius (m) |
| 336 ± 1 | 18.33 ± 0.03 | 0.043 ± 0.003 |
| 360 ± 1 | 18.97 ± 0.03 | 0.045 ± 0.003 |
| 384 ± 1 | 19.59 ± 0.03 | 0.048 ± 0.003 |
| 410 ± 1 | 20.25 ± 0.03 | 0.050 ± 0.003 |
| 430 ± 1 | 20.74 ± 0.03 | 0.053 ± 0.003 |
| 454 ± 1 | 21.30 ± 0.03 | 0.055 ± 0.003 |
| 479 ± 1 | 21.89 ± 0.03 | 0.058 ± 0.003 |

For the second procedure, current was set to a constant 0.90 ± 0.01 A and the Pasco voltage supply required a minimum value of 336 ± 1 V and a maximum value of 479 ± 1 V to produce a neat electron ring. We began to carry out the procedure and the data can be found to the right. Using the data found in this table, we entered it into origin, found values for the radius and plotted these values as a function of the voltage’s root. The slope of the formed line was determined by origin to be (5 ± 0.08) x 10-3 m/V. This linear fit had an intercept of -0.04 ± 0.002 m/V, implying that our data had error. This value should be close to zero as the radius of the electron ring should decrease with a decrease in power supply, but should not reach a negative radius.

The uncertainty in the voltage root can be found by using the derivative method:

(18)

(19)

Using all of these values for the first test and equation 14, we calculated a value for the ratio:

(14)

1.626 x 1011C/Kg

From here, we can calculate the uncertainty in this value:

The final value for the results of the ratio in section two was (1.6 ± 0.1) x 1011 C/Kg. This value makes analytical sense as it is close to the currently accepted value. In fact, the accepted theoretical value for the ratio is within the uncertainty of our measured value. Therefore, our results can be deemed accurate.

A straight line should fit both of these graphs as the vertical and horizontal axes are directly proportional. The first value for the ratio of electron charge to mass was comparatively much worse than the value determined by the second section. Both values uncertainties included the accepted theoretical value into its range. However, section two surely provided a lesser uncertainty and a value substantially closer to the theoretical ratio value.

**Conclusions:**

The purpose of this lab was to find the ratio of an electron to its mass over the course of two different experiments, and observe the changes in radius in two separate procedures with a changing current and power supply. For the data discussed and obtained, all parts were averaged between my lab partner and me. We chose to average the data for diameters because we agreed neither one of us were going to be better at measuring from the distance necessary to encapsulate the mirrored electron ring and under such dark conditions. From the first graph illustrating the linear relationship between an inverted radius for the electron ring and a changing current, the slope of the linear fit was 16 ± 3 m-1/A. This value corresponded to a value of (2.5 ± 0.9) x 1011 C/kg for the charge to mass ratio. Our data was very far from being linear. Origin was able to create a decent linear fit, but this linear fit is most likely very unreliable, like the data points we took. The intercept for this graph was calculated by Origin to be 20 ± 4 m-1/A, implying that our data was already skewed from the beginning. This value should be close to zero because as the supplied current is reduced, the radius of the electron ring should increase to an undefined number (radius = 1/0 m and radius inverted = 0/1 m-1). Our intercept value is unreasonable for this experiment as there is not a great enough external force to create such an error in results. From the second graph illustrating the linear relationship between a radius for the electron ring and a changing voltage root, the slope of the linear fit was (5 ± 0.08) x 10-3 m/V. This value corresponded to a value of (1.6 ± 0.1) x 1011 C/kg for the charge to mass ratio. The graph of this attempt to find the ratio of charge to mass is superb compared to the first graph. The linear fit is almost perfect considering our data points. The linear fit for graph two projected an intercept value of -0.04 ± 0.002 m/V. This value should again be zero meters per volt because the radius should decrease as a direct result of a voltage decrease as seen during the procedure. This section’s intercept is much smaller than the first section’s intercept, comparatively to the rest of the results, further proving this is the more accurate of the two tests for our lab group. My lab partner and I chose this result because of its accuracy as opposed to the first ratio calculated. The uncertainty in the first part of lab was much greater as compared to the second part of lab. This is probably because we gained much experience in taking measurements and provided better results. The value obtained from the radius versus voltage root graph was within the uncertainty of the accepted ratio which is (1.75882017 +/– 0.00000007) x 1011 C/kg. Our values didn’t perfectly match with the theoretical ratio because of error in the procedure and tools used. The system for measuring the diameter of the ring was absolutely awful. It included standing a distance away from the apparatus and encompassing the mirrored view of the ring within the actual view. However, the mirrored view was extremely hard to find. It proved so difficult that our error in this portion alone, accounted for most the error. Also with the same system, it was hard to measure the individual lines on the ruler because of the distance away one needs to stand and the various random reflections on the reflective ruler. All of this may be contributed by the poor eyesight of my lab partner and me. The last major role in uncertainty was accounted for by the wideness of the beam. Even with the tuner to changer the size of the beam width, the beam still measured almost a quarter centimeter, vastly contributing to overall uncertainty. The uncertainty in this lab can be reduced by decreasing the ambient light in the room. The darker the room is, the less reflections on the mirror, and the mirrored image of the ring will be easier to see. A better method for measuring diameter is a good general improvement upon this lab. In conclusion, our data allowed us to calculate at least one reasonable value for the ratio of electric charge of an electron to its mass.

**Acknowledgements:**

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**References:**

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